

# Weibull Analysis of HP Packings of the Secondary Compressor in the Aryasasol LDPE Plant

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## Abstract

The objective of this research is to calculate the optimal time for performing preventive maintenance on the HP Packings<sup>2</sup> of the secondary compressor in the Low-Density Polyethylene (LDPE) unit of Aryasasol Polymer Company, using Weibull analysis. In this study, the life data of HP Packings from 6 cylinders in the second stage of this compressor were used, covering an 8-year period from 2009 to 2016. To verify the life data, the Laplace Trend Test<sup>3</sup> was applied. After calculating the Weibull distribution parameters, the Kolmogorov–Smirnov<sup>4</sup> test was performed to confirm their statistical significance. Subsequently, a cost model was developed to calculate the optimal preventive maintenance interval.

The results indicate that for one packing, Weibull analysis could not be performed due to a trend in the life data. The remaining four HP packings exhibited a slightly increasing failure pattern, and for one cylinder the cost optimization function was constructed and solved.

**Keywords:** Weibull Analysis, Cost Function, Laplace Trend Test, Kolmogorov–Smirnov Test

## Introduction

In 1951, Swedish engineer and mathematician Waloddi Weibull<sup>5</sup> proposed a mathematical model capable of describing the failure behaviour of many components. This model is a probability distribution function defined by two parameters  $\beta$  and  $\eta$ , as shown in Equation 1.

$$f(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} \exp \left[ - \left( \frac{t}{\eta} \right)^\beta \right] \text{ for } t \geq 0$$

(1) Weibull Probability Density Function

One strength of this distribution is that by varying  $\beta$  and  $\eta$  it can approximate a wide range of distributions, including Normal, Exponential, and Hyper-exponential.

After World War II, as commercial aviation expanded rapidly, the crash rate rose to approximately 40 aircraft per million flights. Had this trend continued, a Boeing 747 would have crashed in the news every day [1]. Airlines initially tried reducing time between overhauls, but this approach failed — and sometimes increased failure rates. In 1968 a team called MSG1<sup>7</sup> at United Airlines (UAL)<sup>8</sup> analysed aircraft component failure patterns and discovered 6 distinct failure patterns. These patterns correspond to the Hazard Functions<sup>9</sup> of the Weibull distribution and later became the foundation of Reliability-Centred Maintenance<sup>10</sup> across many industries.



Figure 1 – Six Failure Patterns (UAL 1968 / Bromberg 1973 / US Navy 1982)

Contrary to the traditional view, 89% of aircraft components in the American study showed no direct relationship between failure probability and age (patterns D, E, F — combined 7%+14%+68% in Figure 1). Similar findings emerged from the Swedish Bromberg study (1973) and the U.S. Navy<sup>11</sup> research (1982) [2].

These findings transformed maintenance strategy. Today, aircraft are the most reliable form of transport — averaging 0.01 crashes per million flights, a 400% improvement attributed to Weibull-based maintenance strategy selection.

Pattern C shows a monotonically increasing hazard with no distinct wear-out region, making it impossible to define an optimal PM interval [1]. Its  $\beta$  is slightly above 1. Patterns A and B have  $\beta > 1$  with a sharp increase in failure probability, allowing calculation of an optimal PM interval. Pattern E has  $\beta = 1$  (constant hazard); Pattern F has  $\beta < 1$  (decreasing hazard).

The secondary compressor at Aryasasol is a two-stage reciprocating unit with 12 cylinders, raising ethylene pressure from 250 bar (first stage) to 1,180 bar, and then to 2,780 bar (second stage) for reactor injection. The HP Packing is a high-pressure mechanical seal that prevents gas leakage while the plunger compresses the gas.

The secondary compressor is the most critical piece of equipment in the unit — its failure forces a complete shutdown of the LDPE plant. Each HP Packing replacement costs over 70 million Tomans including lost production, labour, and parts; if upstream units such

as the Olefin unit are also affected, this cost can double or triple. Optimising the PM interval can therefore significantly improve company profitability.

### Main Body

Since a minimum of 5 life data points is required for Weibull analysis, this study used life data from the second-stage HP Packings of cylinders tagged 2A, 2B, 2C, 2D, 2E, and 2F, all of which had sufficient data. These cylinders are highlighted with red arrows in Figure 2.

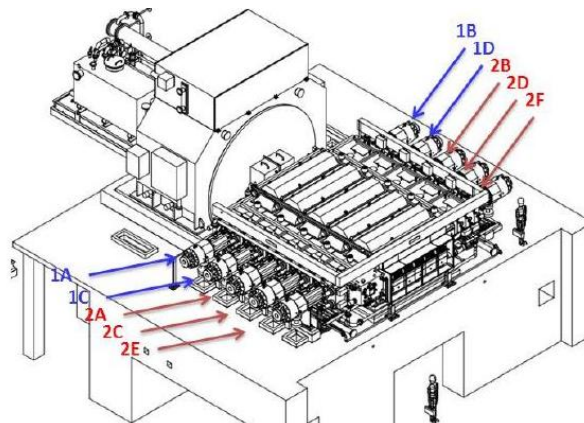


Figure 2 – Secondary Compressor Schematic with Cylinder Numbers (Blue arrows: 1st stage cylinders 1B, 1D, 2B, 2D, 2F; Red arrows: 2nd stage cylinders 1A, 1C, 2A, 2C, 2E)

A key requirement for Weibull analysis is that the life data must be iid<sup>12</sup> (Independently and Identically Distributed) — i.e. the failure time distribution must be the same each cycle, with no systematic trend (improving or degrading). The Laplace Trend Test was applied to verify this. The test statistic is:

$t_i$  = life of the component at the  $i$ -th failure

$N(t_n)$  = total number of failures up to time  $t_n$

$T$  = total cumulative operating time of the equipment from start to present

$$u = \sqrt{12N(t_n)} \left( \frac{\sum_{i=1}^n t_i}{T \cdot N(t_n)} - 0.5 \right) \quad (2)$$

(2)

If  $u$  follows a standard Normal distribution (mean 0, standard deviation 1), the null hypothesis of iid data cannot be rejected.



Figure 3 – Life Data of HP Packing Cylinder 2D at Each Repair

For HP Packing 2D, the Laplace statistic  $u = -2.32379$ , which falls below the critical value of  $-1.96$ , placing it outside the acceptable range. At 95% confidence, reliability has been improving over time — a positive trend exists. Therefore this packing's life data cannot be used for Weibull analysis.

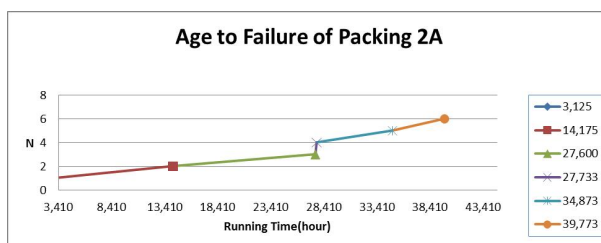


Figure 4 – Life Data of HP Packing Cylinder 2A at Each Repair

For HP Packing 2A,  $u = 0.314$ , which lies within the  $\pm 1.96$  range. At 95% confidence, the null hypothesis of iid data cannot be rejected — no systematic trend is observed. This data can therefore be used for Weibull analysis.

After confirming the absence of trend, the Weibull parameters were calculated using the Weibull Plot<sup>13</sup> and Median Rank<sup>14</sup> method in OREST software. For calculation details see [3]. The resulting parameters for Packing 2A are  $\beta = 1.19$  and  $\eta = 28,911$ .

To verify that these parameters are statistically significant, the Kolmogorov–Smirnov test was applied:


$$d = \max_i (|F(t_i) - \hat{F}(t_i)|, |F(t_i) - \hat{F}(t_{i-1})|) \quad (3)$$

(3)

where  $\hat{F}(t)$  is the empirical CDF from sample data and  $F(t)$  is the CDF from the fitted Weibull distribution. If the calculated  $d$  exceeds  $d_\alpha$  from the K-S table, the null hypothesis of good fit is rejected.

For Packing 2A,  $d = 0.346$  and  $d_\alpha = 0.519$ . Since  $d < d_\alpha$ , at 95% confidence the Weibull fit to the life data cannot be rejected.

Table 1 – Weibull Analysis Results for HP Packings in the Second Stage

No	Packing	iid Test(5% significant level)	KS Test(5% significant level)	Shape $\beta$	Scale $\eta$	Hazard Function
1	2A	OK	OK	1.19	28911	
2	2B	OK	OK	1.19	25364	
3	2C	OK	OK	1.58	26873	
4	2D	NOK(reliability growth)				
5	2E	OK	OK	1.05	37808	
6	2F	OK	OK	1.90	32123	

Note: NOK = Not OK (reliability growth trend detected; Weibull analysis not applicable)

From Table 1, HP Packings 2A, 2B, 2C, and 2E all have  $\beta \approx 1$ , meaning failure probability increases only gradually with age. Since there is no sharp wear-out inflection, an optimal PM interval cannot be calculated for these packings — Condition-Based Maintenance (CBM)<sup>18</sup> remains appropriate. For Packing 2F,  $\beta = 1.90$  (close to 2), suggesting a PM interval may be calculable.

Define  $t_p$  as the duration after which preventive maintenance is performed. If the component fails before  $t_p$ , corrective repair is performed and the clock resets. The objective is to minimise total cost — preventive cost  $C_p$  plus corrective cost  $C_r$ .

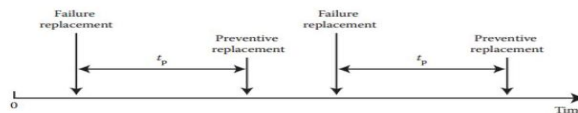


Figure 5 – Problem Schematic

In simple terms, the mathematical model is the ratio of the weighted-average expected cost per cycle to the weighted-average expected cycle length:

$$C(t_p) = \frac{\text{Total expected replacement cost per cycle}}{\text{Expected cycle length}} \quad (4)$$

(4)

The extended form of the cost model is:

$$C(t_p) = \frac{C_p R(t_p) + C_f (1 - R(t_p))}{t_p R(t_p) + \int_0^{t_p} t f(t) dt} \quad (5)$$

(5)

where  $f(t)$  is the Weibull PDF from Equation (1), and  $R(t)$  is the Weibull Reliability Function<sup>15</sup>:

$$R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^\beta\right] \quad (6)$$

(6)

Repair costs include plunger reconditioning, HP Packing, Central Valve<sup>16</sup>, LP Packing, depreciation, labour, and lost production. In a corrective failure the plunger and HP Packing may be severely damaged and unfit for reuse. In a preventive repair they are typically undamaged and can be polished and reused — up to 3–4 times before dimensional tolerances are exceeded.

Since lost production constitutes over 70% of total costs, and the incremental difference in material costs between corrective and preventive repair is small,  $C_p \approx C_f$  is a reasonable approximation. The model simplifies to:

$$C(t_p) = \frac{C_f}{t_p e^{-(t_p/\eta)^\beta} + \int_0^{t_p} t_p [\beta/\eta] (t_p/\eta)^{\beta-1} e^{-(t_p/\eta)^\beta} dt} \quad (7)$$

(7)

Substituting  $\beta = 1.9$ ,  $\eta = 32,123$ , and  $C_f = 7,151,329,405$  and solving in MATLAB yields the following cost curve:

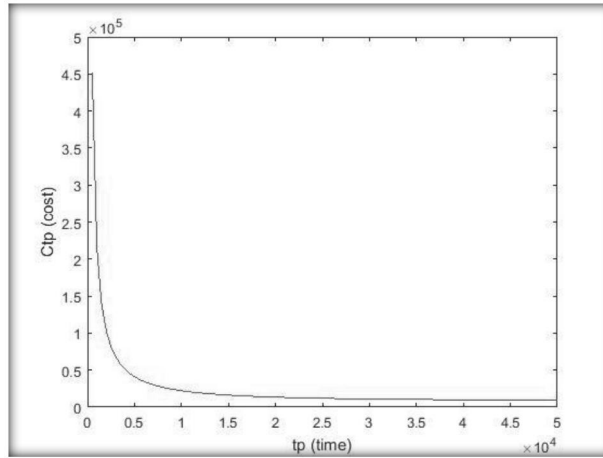


Figure 6 – Cost Function  $C(t_p)$  for HP Packing 2F Repair in the Secondary Compressor

As the curve shows, there is no minimum — the cost function decreases monotonically and never turns up. Consequently an optimal PM interval cannot be identified for Packing 2F either. As a next step, the Proportional Hazards Model (PHM)<sup>17</sup> could be applied, incorporating condition-monitoring parameters such as temperature, vibration, and gas leakage volume.

### Conclusions and Summary

1. For HP Packings 2A, 2B, 2C, and 2E,  $\beta$  is close to 1 with no sharp wear-out inflection. An optimal PM interval cannot be calculated; the current Condition-Based Maintenance (CBM)<sup>18</sup> strategy is appropriate.
2. For HP Packing 2F, although  $\beta = 1.90$  (close to 2), the cost function has no minimum and no optimal PM interval can be determined. CBM remains the best available strategy.
3. Use of advanced condition-monitoring techniques such as the Proportional Hazards Model (PHM), incorporating parameters such as temperature, vibration, and gas leakage volume, is recommended for future analysis.

### Acknowledgments

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### References

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<sup>1</sup> Weibull Analysis   <sup>2</sup> High Pressure Packing   <sup>3</sup> Laplace Trend Test   <sup>4</sup> Kolmogorov–Smirnov   <sup>5</sup> Waloddi Weibull

<sup>6</sup> PDF of Weibull Distribution   <sup>7</sup> Maintenance Steering Group   <sup>8</sup> United Airlines (UAL)   <sup>9</sup> Hazard Function   <sup>10</sup> Reliability-Centred Maintenance

<sup>11</sup> US Navy   <sup>12</sup> Independently Identically Distributed   <sup>13</sup> Weibull Plot   <sup>14</sup> Median Rank   <sup>15</sup> Reliability Function

<sup>16</sup> Central Valve   <sup>17</sup> Proportional Hazards Model   <sup>18</sup> Condition-Based Maintenance